

### 11-5 Square Roots of Variable Expressions

**Objective:** To find square roots of variable expressions and to use them to solve equations and problems.

**Property**

**Property of Square Roots of Equal Numbers** For any real numbers  $r$  and  $s$ :  
 $r^2 = s^2$  if and only if  $r = s$  or  $r = -s$ .

**CAUTION** When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example,  $\sqrt{x^2} = |x|$ , not  $x$ .

**Example 1** Simplify: a.  $\sqrt{144x^2}$  b.  $\sqrt{25n^8}$  c.  $\sqrt{12a^3}$

**Solution**

a.  $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$   
 $= 12|x|$

b.  $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$   
 $= \sqrt{25} \cdot \sqrt{(n^4)^2}$   
 $= 5n^4$  ( $n^4$  is always nonnegative)

c.  $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$   
 $= \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$   
 $= 2|a|\sqrt{3a}$

**Simplify.**

- |  |   |   |
|--|---|---|
| 1. $\sqrt{81x^2} \ 9 x $                               | 2. $\sqrt{121x^2} \ 11 x $                            | 3. $\sqrt{20x^2} \ 2\sqrt{5} x $                                  |
| 4. $\sqrt{45x^4} \ 3\sqrt{5}x^2$                       | 5. $-\sqrt{25x^2} \ -5 x $                            | 6. $-\sqrt{16c^4} \ -4c^2$  |
| 7. $-\sqrt{64d^8} \ -8d^4$                             | 8. $-\sqrt{98n^6} = 7\sqrt{2} n^3 $                   | 9. $\sqrt{225y^4} \ 15y^2$  |
| 10. $\sqrt{400a^6b^4} \ 20 a^3 b^2$                    | 11. $\sqrt{81m^{12}} \ 9m^6$                          | 12. $\sqrt{441n^6} \ 21 n^3 $                                     |
| 13. $\pm\sqrt{75x^2y^3} \pm 5 xy \sqrt{3y}$            | 14. $\pm\sqrt{60x^6y^4} \pm 2 x^3 y^2\sqrt{15}$       | 15. $-\sqrt{121x^2y^2} \ -11 xy $                                 |
| 16. $-\sqrt{900a^4b^6} \ -30a^2 b^3 $                  | 17. $\pm\sqrt{\frac{81x^8}{100}} \pm \frac{9}{10}x^4$ | 18. $\pm\sqrt{\frac{121}{225x^{10}}} \pm \frac{11}{15 x^5 }$      |
| 19. $\sqrt{\frac{x^4y^8}{9z^2}} \ \frac{x^2y^4}{3 z }$ | 20. $\sqrt{\frac{32m^3n^2}{2mn^2}} \ 4 m $            | 21. $\sqrt{\frac{16x^{18}}{3600y^{20}}} \ \frac{ x^9 }{15y^{10}}$ |
| 22. $\sqrt{\frac{256x^{40}}{25}} \ \frac{16x^{20}}{5}$ | 23. $\sqrt{2.25x^4} \ 1.5x^2$                         | 24. $-\sqrt{2.56k^2} \ -1.6 k $                                   |

### 11-5 Square Roots of Variable Expressions (continued)

**Example 2** Simplify  $\sqrt{m^2 - 8m + 16}$ .

**Solution**  $\sqrt{m^2 - 8m + 16} = \sqrt{(m - 4)^2} = |m - 4|$

**Simplify.**

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|-------------------------------------|---------------------------------------|
| 25. $\sqrt{x^2 + 4x + 4} \  x + 2 $ | 26. $\sqrt{n^2 - 14n + 49} \  n - 7 $ |
| 27. $\sqrt{x^2 - 6x + 9} \  x - 3 $ | 28. $\sqrt{m^2 - 10m + 25} \  m - 5 $ |

**Example 3** Solve  $4x^2 = 25$ .

<p><b>Solution 1</b></p> $4x^2 = 25$ $4x^2 - 25 = 0$ $(2x + 5)(2x - 5) = 0$ $2x = -5 \quad \text{or} \quad 2x = 5$ $x = -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2}$ <p>Check: <math>4\left(\frac{5}{2}\right)^2 \stackrel{?}{=} 25</math>      and      <math>4\left(-\frac{5}{2}\right)^2 \stackrel{?}{=} 25</math></p> $25 = 25 \checkmark \quad \text{and} \quad 25 = 25 \checkmark$ <p>The solution set is <math>\left\{\frac{5}{2}, -\frac{5}{2}\right\}</math>.</p>	<p><b>Solution 2</b> <math>4x^2 = 25</math></p> $x^2 = \frac{25}{4}$ $x = \pm\sqrt{\frac{25}{4}}$ $x = \pm\frac{5}{2}$
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**Solve.**

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|--|--|--|--|
| 29. $x^2 = 16 \ \{-4, 4\}$                 | 30. $n^2 = 36 \ \{-6, 6\}$                 | 31. $x^2 - 9 = 0$                          | 32. $a^2 - 25 = 0$                         |
| 33. $0 = a^2 - 49$                         | 34. $0 = m^2 - 64$                         | 35. $2m^2 - 18 = 0$                        | 36. $40b^2 - 160 = 0$                      |
| 37. $36y^2 - 16 = 0$                       | 38. $4c^2 - 25 = 0$                        | 39. $0 = 49z^2 - 9$                        | 40. $0 = 45x^2 - 125$                      |
| $\left\{-\frac{2}{3}, \frac{2}{3}\right\}$ | $\left\{-\frac{5}{2}, \frac{5}{2}\right\}$ | $\left\{-\frac{3}{7}, \frac{3}{7}\right\}$ | $\left\{-\frac{5}{3}, \frac{5}{3}\right\}$ |

**Mixed Review Exercises**

**Simplify.**

- |                                     |                                |                                    |
|-------------------------------------|--------------------------------|------------------------------------|
| 1. $\pm\sqrt{80} \pm 4\sqrt{5}$     | 2. $-4\sqrt{75} \ -20\sqrt{3}$ | 3. $3\sqrt{256} \ 48$              |
| 4. $2^{-3} - 3^{-2} \ \frac{1}{72}$ | 5. $4^3 \cdot 2^{-5} \ 2$      | 6. $(3x^2)^3(-2x^4)^2 \ 108x^{14}$ |

**Evaluate if  $x = 9$ ,  $y = 16$ , and  $n = 1$ .**

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|-----------------------|------------------|-----------------------|------------------------------|--|-------------------------|
| 7. $x^2 + y^2$<br>337 | 8. $x^2n^2 \ 81$ | 9. $y^2 - x^2$<br>175 | 10. $\sqrt{\frac{y}{n}} \ 4$ | 11. $\sqrt{\frac{x}{y}} \ \frac{3}{4}$ | 12. $(\sqrt{y})^2 \ 16$ |
|-----------------------|------------------|-----------------------|------------------------------|--|-------------------------|